Physics 101H

General Physics 1 – Honors

Lecture 7 – 9/12/22

2D motion

Problem sets

Problem Set 1 is due by the **start of class** on **Wednesday 14 September**

Remember

- There are two components – online and handwritten
- I will **drop the lowest grade** on your weekly Problem Sets

MAKING SURE YOUR WORK IS LEGIBLE IS *YOUR* RESPONSIBILITY

Summary

Topics

Last week

- Vectors
- Kinematics in 1D
- Kinematics in 2D

This week

- 2D motion
- Circular motion
- Forces

Today: kinematics in 2D [chapter 4]

- Describing motion in 2D
- Position, velocity, acceleration
- Constant acceleration
- Projectile motion

Announcements

Wednesday: Problem set 1 due
Problem set 2 assigned

Multiple choice

Instructions: Consider the following question. After you have had a chance to think, I will ask you to raise your hands to indicate your answer.

Questions: 1. Can velocity be constant while acceleration is nonzero?

(a) yes (b) no

2. What about speed?

(a) yes (b) no

3. Does motion in one direction affect the motion in a perpendicular direction?

2D motion

Motion gets a little more interesting

Acceleration

Acceleration is the rate of change of velocity

But velocity is a vector and can therefore change in two ways!

- Magnitude can change
- Direction can change

Constant acceleration in 2D

Motion in perpendicular directions is **totally independent**!

Write vectors in terms of components along perpendicular directions and treat them completely separately

Our 1D equations from Chapter 3 apply to each component separately

Projectile motion

A **projectile** will follow a **parabolic trajectory**

The distance travelled is the **range**

The maximum height reached is the **height**

Practice in pairs

Instructions: Discuss the following question with a neighbour. Your answers will not be graded; your discussion is for your own learning*.

Question: A cart on a roller-coaster rolls down the track in the figure. As the cart rolls past the point shown, what happens to its speed and acceleration in the direction of motion?

(a) Both decrease

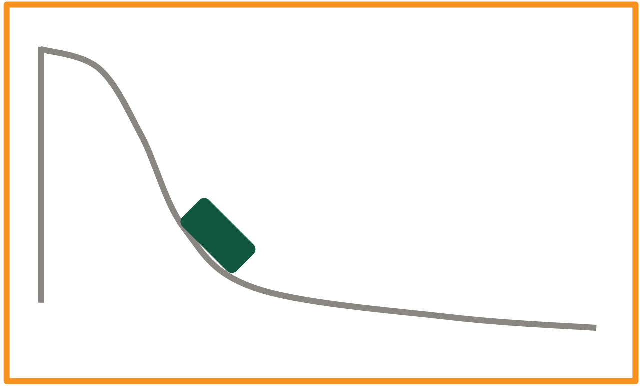
(b) Speed decreases, but acceleration increases

(c) Both remain constant

(d) Speed increases, but acceleration decreases

(e) Both increase

(6) None of the above



Example: A mountain lions can jump a height of 4 m, when leaving the ground at an angle of 45 degrees. At what speed does it leave the ground to make this jump?

Summary

Topics

This week

- 2D motion
- Circular motion
- Forces

Today: kinematics in 2D [chapter 4]

- Constant acceleration
- Projectile motion

Wednesday: kinematics in 2D

- Examples in 2D
- Uniform circular motion

Announcements

Wednesday: Problem set 1 due
Problem set 2 assigned

NEXT WEEK:
NO CLASS ON THURSDAY OR FRIDAY

PHYSICS 101 - HONORS

Lecture 7 9/12/22

2D acceleration (slide 6)

$$\bar{r}(t) = x\hat{\imath} + y\hat{\jmath}$$

$$\bar{v}(t) = \frac{d\bar{r}}{dt} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} = v_x\hat{\imath} + v_y\hat{\jmath}$$

$$\bar{a}(t) = \frac{d\bar{v}}{dt} = \frac{d^2\bar{r}}{dt^2} = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath} = \frac{d^2x}{dt^2}\hat{\imath} + \frac{d^2y}{dt^2}\hat{\jmath}$$

$$= a_x\hat{\imath} + a_y\hat{\jmath}$$

Kinematic components (slide 7)

$$v_x = a_x t + v_{ox}$$
$$x = \frac{1}{2}a_x t^2 + v_{ox}t + x_o$$

$$v_y = a_y t + v_{oy}$$
$$y = \frac{1}{2}a_y t^2 + v_{oy}t + y_o$$

Or

$$\bar{v} = \bar{a}t + \bar{v}_o$$
$$\bar{r} = \frac{1}{2}\bar{a}t^2 + \bar{v}_o t + \bar{r}_o$$

$\leftarrow$

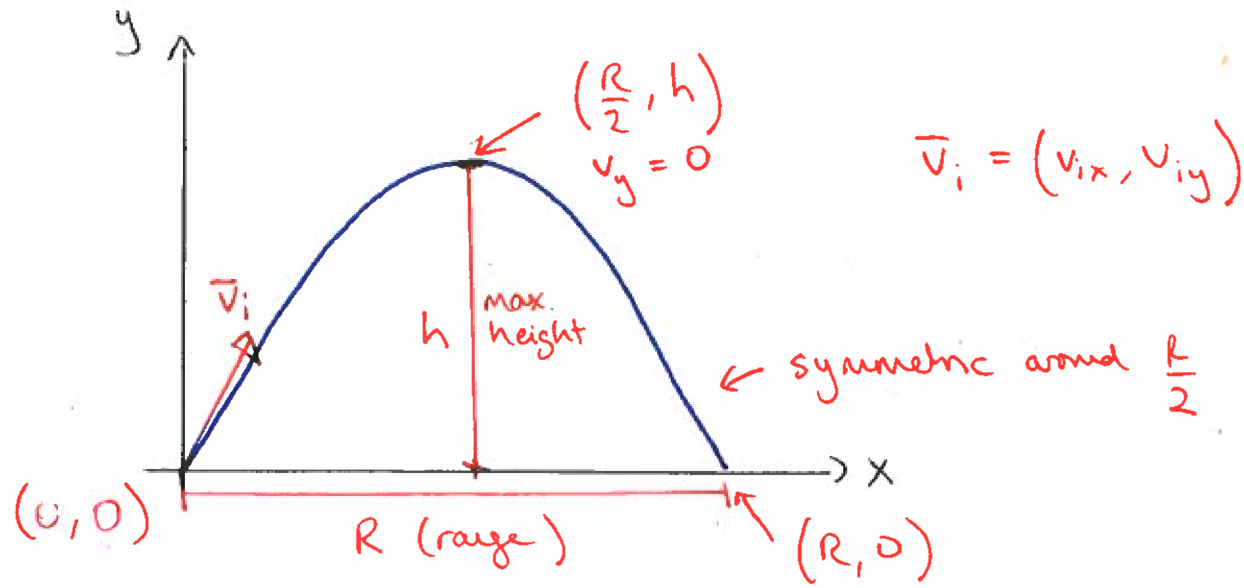
$$\bar{v} = v_x\hat{\imath} + v_y\hat{\jmath}$$
$$= (a_x t + v_{ox})\hat{\imath} + (a_y t + v_{oy})\hat{\jmath}$$
$$= \underbrace{(a_x\hat{\imath} + a_y\hat{\jmath})}_{\bar{a}}t + \underbrace{v_{ox}\hat{\imath} + v_{oy}\hat{\jmath}}_{\bar{v}_o}$$

Projectile motion (slide 8)

Projectile

acceleration due to gravity (y direction)
no acceleration in x direction

Draw



Projectile motion

Motion in x direction: $v = v_{ix}$ $(a_x = 0)$

$x = v_{ix} t$ → "time of flight" is $T = \dfrac{R}{v_{ix}}$

In y direction: $y = -\frac{1}{2} g t^2 + v_{iy} t + y_0$

$$y = -\frac{1}{2} g \left(\frac{x}{v_{ix}}\right)^2 + v_{iy} \frac{x}{v_{ix}} + y_0$$

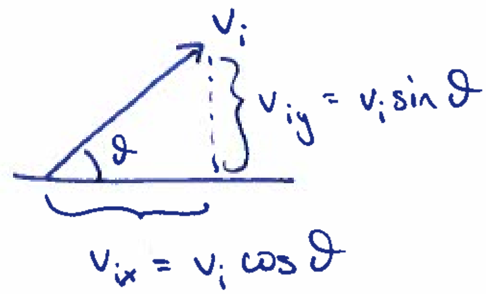
$$y = -\frac{1}{2} \frac{g}{v_{ix}^2} x^2 + \frac{v_{iy}}{v_{ix}} x + y_0$$

equation for a parabola (a quadratic!)

Projectile motion (slide 8)

Launch projectile at an angle θ to ground

$\bar{v}_i = v_i \cos\theta \, \hat{\imath} + v_i \sin\theta \, \hat{\jmath}$



$$\Rightarrow y(x) = -\frac{1}{2} \frac{g}{(v_i \cos\theta)^2} x^2 + \frac{v_i \sin\theta}{v_i \cos\theta} x + \overset{=0}{\cancel{y_0}}$$

$$= -\frac{g}{2v_i^2 \cos^2\theta} x^2 + \frac{\sin\theta}{\cos\theta} x$$

"Complete the square"

$$y(x) = -\frac{g}{2v_i^2 \cos^2\theta} \left[x^2 - \frac{2v_i^2 \sin\theta \cos\theta}{g} x + \left(\frac{v_i^2 \sin\theta \cos\theta}{g}\right)^2 - \left(\frac{v_i^2 \sin\theta \cos\theta}{g}\right)^2 \right]$$

$$= -\frac{g}{2v_i^2 \cos^2\theta} \left[x - \frac{v_i^2 \sin\theta \cos\theta}{g} \right]^2 + \frac{v_i^2 \sin^2\theta}{2g}$$

Projectile motion

Derived $y(x) = \frac{-g}{2v_i^2 \cos^2\theta} \left[x - \frac{v_i^2 \sin\theta \cos\theta}{g} \right]^2 + \frac{v_i^2 \sin^2\theta}{2g}$

Compare $y = c(x - x_v)^2 + y_v \leftarrow$ parabola with vertex (x_v, y_v)

Note $x_v = \frac{R}{2} = \frac{v_i^2 \sin\theta \cos\theta}{g}$ 　　 Range $= \boxed{R = \frac{2v_i^2 \sin\theta \cos\theta}{g}}$

$y_v = h = \frac{v_i^2 \sin^2\theta}{2g}$ 　　 Height $= \boxed{h = \frac{v_i^2 \sin^2\theta}{2g}}$

Range : $R = \frac{2 v_i^2 \sin\theta \cos\theta}{g} = \frac{v_i^2}{g} \sin 2\theta$

$\Rightarrow R_{max} = \frac{v_i^2}{g} (\sin 2\theta)_{max}$

$(\sin 2\theta)_{max} = 1$

$2\theta = 90^\circ$

$\theta = 45^\circ$

$\Rightarrow \boxed{R_{max} = \frac{v_i^2}{g}}$

Height : $h = \frac{v_i^2}{2g} \sin^2\theta$

$(\sin^2\theta)_{max} = 1^2$

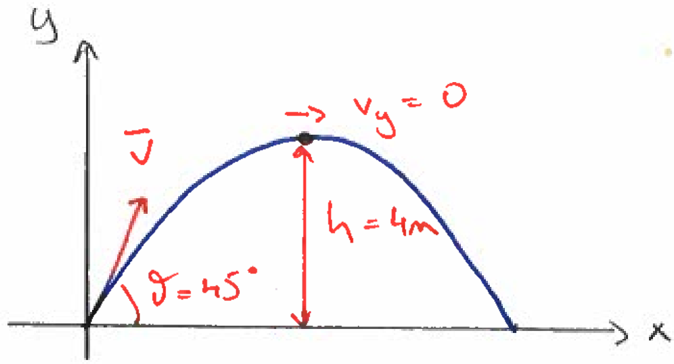
$\sin\theta = 1$

$\theta = 90^\circ$ ← then $R = 0$!

$\Rightarrow \boxed{h_{max} = \frac{v_i^2}{2g}}$

Mountain lion example (slide 9)

Draw a diagram



$$\bar{v} = v\cos\theta\,\hat{i} + v\sin\theta\,\hat{j}$$
$$= v\cos 45°\,\hat{i} + v\sin 45°\,\hat{j}$$
$$= \frac{v_0}{\sqrt{2}}\,\hat{i} + \frac{v_0}{\sqrt{2}}\,\hat{j}$$

Mountain lion example (slide 8)

Max height when $v_y = 0$

$$v_y(t) = -gt + v_{0y} \Rightarrow 0 = -gt + \frac{1}{\sqrt{2}}v \Rightarrow t = \frac{v}{\sqrt{2}g}$$

$$y(t) = -\frac{g}{2}t^2 + v_{0y}t + y_0 \Rightarrow y_{max} = 4m = -\frac{g}{2}\left(\frac{v}{\sqrt{2}g}\right)^2 + \frac{v}{\sqrt{2}}\cdot\left(\frac{v}{\sqrt{2}g}\right)$$

$$= -\frac{g}{2}t^2 + \frac{v}{\sqrt{2}}t$$

$$= -\frac{v^2}{4g} + \frac{v^2}{2g} = \frac{v^2}{4g}$$

$$\Rightarrow v^2 = 4g\,y_{max} \quad \text{or} \quad v = \sqrt{4 \cdot 9.8 \cdot 4}$$

$$\boxed{v = 12.5\ m/s}$$