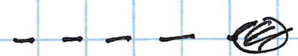
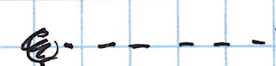
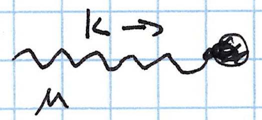
Scattering in Scalar QED

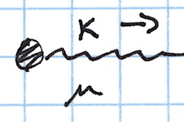
Goals
- [ ] Study Møller scattering in Scalar QED
- [ ] Use Mandelstram variables to make our lives easier

To review, we can break the Feynman rules of Scalar QED into 3 categories

- External states

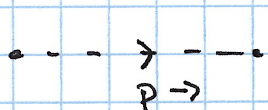
 $= 1$  $= 1$



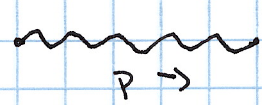
$= \epsilon^{\mu}(k)$ 

$= \epsilon^{*\mu}(k)$

- Propagators

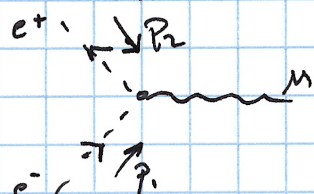


$$= \frac{i}{p^2 - m^2 + i\epsilon}$$

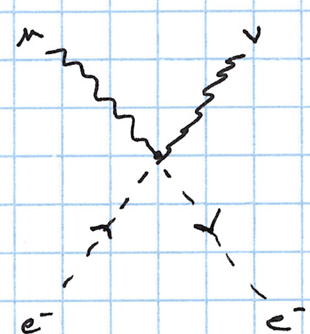


$$= \frac{-i}{p^2 + i\epsilon}\left(g_{\mu\nu} - (1-\xi)\frac{p_\mu p_\nu}{p^2}\right)$$

- Interactions



$$= ie(p_\mu' - p_\mu)$$



$$= 2ie^2 g_{\mu\nu}$$

Scattering in Scalar QED

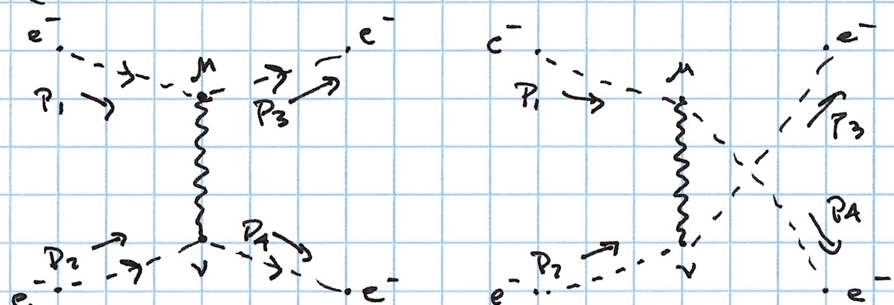
Møller Scattering: $e^-e^- \to e^-e^-$

We'll now use these Feynman rules to study Møller Scattering in Scalar QED.

☆ Why should we care about Møller Scattering?
- related to repulsion of electrons in helium atoms
- electron-electron colliders
- study parity violation in electroweak theory

☆ But we're not using physical ~~fermions~~ electrons, which are fermions. What's the point of this exercise?
- Higgs boson is the only scalar in the Standard Model
- Composite scalars such as the α-particle / helium nucleus and scalar mesons like the Kaon.

Using our Feynman rules, the diagrams for Møller scattering are

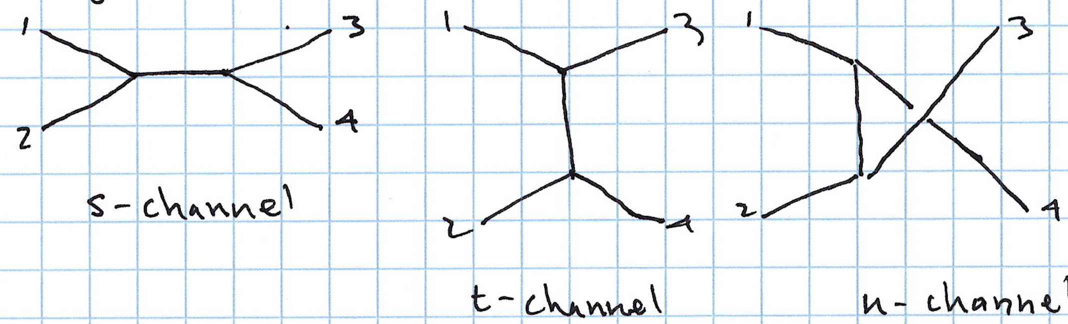


☆ Why are the diagrams different? Would we have two diagrams if we were studying $e^-\mu^- \to e^-\mu^-$?
- Because e^- are identical, we can't tell which final state e^- came from which initial state e^-.
- If we were studying $e^-\mu^- \to e^-\mu^-$, we would always know that the e^- came from the e^- and the μ^- came from the μ^-, so there would only be 1 diagram.

Scattering in Scalar QED

Mandelstram Variables

The forms of these diagrams show up a lot, so we want to give them names. For $2 \to 2$ scattering, we have



We define these forms by the momentum flowing through the intermediate stage.

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2$$

$$t = (p_1 - p_3)^2 = (p_2 - p_4)^2$$

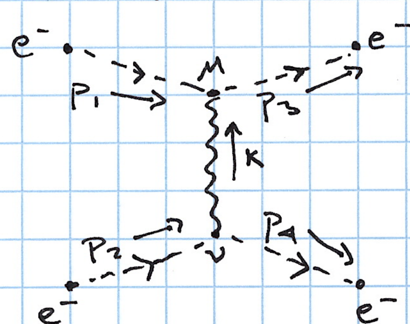
$$u = (p_1 - p_4)^2 = (p_2 - p_3)^2$$

☆ What's nice about Mandelstram variables? Why do we prefer s to $p_m^1 + p_m^2$?

- The Mandelstram variables are Lorentz invariant.

Scattering Amplitude

We can now write down the amplitude for the t-channel



$$= i\mathcal{M}_t = (-ie)(p_1^\mu + p_3^\mu)\left[\frac{-i}{k}\left(g_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2}\right)\right]$$

$$\times (-ie)(p_2^\nu + p_4^\nu)$$

Our momentum transfer is

$$k^\mu = p_3^\mu - p_1^\mu = p_2^\mu - p_4^\mu$$

Scattering in Scalar QED

Scattering Amplitude, continued

Note that $k^2 = (p_1 - p_3)^2 = t$

Consider the gauge fixing term which contains k_μ. Note

$$(p_1^\mu + p_3^\mu) k_\mu = (p_1^\mu + p_3^\mu)(p_3^\mu - p_1^\mu)$$
$$= p_1 \cdot p_3 - p_1^2 + p_3^2 - p_1 \cdot p_3$$
$$= m^2 - m^2 = 0$$

This means that the gauge fixing term will vanish.

So

$$i\mathcal{M}_t = \frac{ie^2}{t} (p_1 + p_3) \cdot (p_2 + p_4)$$

From this, we can quickly find the amplitude of the u-channel diagram by replacing $t \to u$ and $3 \leftrightarrow 4$. Then

$$i\mathcal{M}_u = \frac{ie^2}{u} (p_1 + p_4) \cdot (p_2 + p_3)$$

Cross Section

The cross section for Møller scattering is then

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 E_{cm}} |\mathcal{M}_t + \mathcal{M}_u|^2$$

$$= \frac{e^4}{64\pi^2 s} \left[\frac{(p_1 + p_3) \cdot (p_2 + p_4)}{t} + \frac{(p_1 + p_4) \cdot (p_2 + p_3)}{u} \right]^2$$

Noting that $e^2/4\pi = \alpha$, the fine structure constant, and using our Mandelstram variables, we find

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} \left[\frac{s-u}{t} + \frac{s-t}{u} \right]$$