Extra example: polarised $e^+e^- \to \mu^+\mu^-$

$\boxed{\text{P+S 5.2}}$

This is a very elegant result - but it is unclear where this high energy limit of $(1+\cos^2\theta)$ comes from. To understand this, we need to consider the more complicated case of polarised scattering, in which the e^+e^- have different spin states and we measure the final $\mu^+\mu^-$ spin states.

We will do this two different ways, following P+S, and we will work in the high energy regime to make our lives easier. This means that we have

$$\frac{M_{(\mu)}^2}{E^2} \ll 1 \quad \text{and} \quad \frac{M_{(e)}^2}{E^2} \ll 1.$$

In the massless limit, helicity and chirality coincide, so we can use our chirality projectors to single out specific spin polarisations for our spinors.

Recall that

- $P_{R/L} = \frac{1 \pm \gamma^5}{2} = \begin{cases} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \\ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \end{cases}$

- $i\mathcal{M} = \frac{ig^2}{(p+p')^2} \underbrace{\left(\bar{v}^{s'}(\bar{p}')\gamma^\mu u^s(\bar{p})\right)}_{e^+e^-} \underbrace{\left(\bar{u}^r(\bar{k})\gamma_\mu v^{r'}(\bar{k}')\right)}_{\mu^+\mu^-}$

$\boxed{2.}$

We can project out the $\underset{\hookrightarrow e_R^-}{e^-(\text{hel}=+)}\,\underset{\hookrightarrow e_L^+}{e^+(\text{hel}=-)}$ component using

$$\left(v\cdot\tfrac{1}{2}(1+\gamma^5)\right)^\dagger\gamma^0\gamma^\mu\left(\tfrac{1}{2}(1+\gamma^5)u\right)$$

$$= v^\dagger\tfrac{1}{2}(1+\gamma^5)\gamma^0\gamma^\mu\tfrac{1}{2}(1+\gamma^5)u \qquad \gamma^5 \text{ is Hermitian}$$

$$= v^\dagger\gamma_0\tfrac{1}{2}(1-\gamma^5)\gamma^\mu\tfrac{1}{2}(1+\gamma^5)u$$

$$= \bar{v}\gamma^\mu\tfrac{1}{2}(1+\gamma^5)\tfrac{1}{2}(1+\gamma^5)u \qquad \text{recall } P_R^2 = P_R\ !$$

$$= \bar{v}\gamma^\mu\tfrac{1}{2}(1+\gamma^5)u$$

↑ N.B. This projector ensures both u and $\bar{v}$ are right-handed. <u>But</u> $\bar{v}_R$ is a left-handed positron! So we have picked out a right-handed electron ($h=+$) and a left-handed positron ($h=-$). The amplitude vanishes if both are left-handed (this can be traced back to $P_R P_L = 0$).

We also need the $\mu^-(\text{hel}=-)\ \mu^+(\text{hel}=+)$ piece

$e_{\mu_R^-}$ $\qquad$ $e_{\mu_L^+}$

$$\bar{u}\ \gamma_\nu \tfrac{1}{2}(1+\gamma^5)v$$

$$\Rightarrow\ i\mathcal{M} = \frac{ig^2}{(p+p')^2}\left(\bar{v}^{s'}\gamma^\mu \tfrac{1}{2}(1+\gamma^5)u^s\right)\left(\bar{u}^r\gamma_\mu \tfrac{1}{2}(1+\gamma^5)v^{r'}\right)$$

Recall that the squared invariant matrix element had two independent traces - over the electron and muon separately

Electron piece is $\qquad$ this <u>is</u> complex conjugation

$$\sum_{ss'}\left(\bar{v}^{s'}(\bar{p}')\gamma^\mu\tfrac{1}{2}(1+\gamma^5)u^s(\bar{p})\right)^*\left(\bar{v}^{s'}(\bar{p}')\gamma^\nu\tfrac{1}{2}(1+\gamma^5)u^s(\bar{p})\right)$$

formality because of projection

$$= \sum_{ss'} u_s^\dagger(\bar{p})\tfrac{1}{2}(1+\gamma^5)\gamma^{\mu\dagger}\gamma^0 v^{s'}(\bar{p}') \times \bar{v}^{s'}(\bar{p}')\gamma^\nu\tfrac{1}{2}(1+\gamma^5)u^s(\bar{p})$$

To get P+S eq 5.1a use $(\bar{v}\gamma^\mu u)^* = \left((\bar{v}\gamma^\mu u)^T\right)^\dagger$

$$= \sum_{s,s'}\bar{u}^s(\bar{p})\tfrac{1}{2}(1-\gamma^5)\gamma^\mu v^{s'}(\bar{p}')\ \bar{v}^{s'}(\bar{p}')\gamma^\nu\tfrac{1}{2}(1+\gamma^5)u^s(\bar{p})$$

$$= \left(\sum_s \bar{u}^s(\bar{p})u^s(\bar{p})\right)\tfrac{1}{2}(1-\gamma^5)\gamma^\mu\left(\sum_{s'} v^{s'}(\bar{p}')\bar{v}^{s'}(\bar{p}')\right)\gamma^\nu\tfrac{1}{2}(1+\gamma^5)$$

$$= \mathrm{Tr}\left[(\not{p}+m_e)\tfrac{1}{2}(1-\gamma^5)\gamma^\mu(\not{p}'-m_e)\gamma^\nu\tfrac{1}{2}(1+\gamma^5)\right]$$

& cycling and taking $\frac{m_e}{|p|} \ll 1$

$$= \mathrm{Tr}\left[\gamma^\mu\not{p}'\gamma^\nu\tfrac{1}{2}(1+\gamma^5)\not{p}\tfrac{1}{2}(1-\gamma^5)\right]$$

$$= \mathrm{Tr}\left[\gamma^\mu\not{p}'\gamma^\nu\not{p}\tfrac{1}{2}(1-\gamma^5)\tfrac{1}{2}(1-\gamma^5)\right] \qquad P_L^2 = P_L$$

&

$$= \mathrm{Tr}\left[\gamma^\mu\not{p}'\gamma^\nu\not{p}\tfrac{1}{2}(1-\gamma^5)\right]$$

$$= \tfrac{1}{2}\mathrm{Tr}\left[\gamma^\mu\gamma^\alpha\gamma^\nu\gamma^\beta\right]p'_\alpha p_\beta - \tfrac{1}{2}\mathrm{Tr}\left[\gamma^\mu\gamma^\alpha\gamma^\nu\gamma^\beta\gamma^5\right]p'_\alpha p_\beta$$

$$= 2\left(p'^\mu p^\nu + p'^\nu p^\mu - g^{\mu\nu}p\cdot p'\right) - 2i\epsilon^{\mu\alpha\nu\beta}p'_\alpha p_\beta$$

The muon piece is

$$\sum_{r,r'} \bar{v}^{r'} \tfrac{1}{2}(1-\gamma^5)\gamma_\mu u^r \; \bar{u}^r \gamma_\nu \tfrac{1}{2}(1+\gamma^5) v^{r'}$$

$$= \mathrm{Tr}\left[\tfrac{1}{2}(1-\gamma^5)\gamma_\mu \not{k} \gamma_\nu \tfrac{1}{2}(1+\gamma^5)\not{k}'\right]$$

$$= \mathrm{Tr}\left[\gamma_\mu \not{k} \gamma_\nu \not{k}' \tfrac{1}{2}(1-\gamma^5)\right]$$

N.B. this what we had for the electron piece, with $p' \to k$, $p \to k'$

$$= 2\left(k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu} k\cdot k'\right) - 2i\epsilon_{\mu\alpha\nu\beta} k^\alpha k'^\beta$$

Putting these pieces together we have

$$|M(e^+_L e^-_R \to \mu^+_L \mu^-_R)|^2 = \frac{g^4}{(p+p')^4} \cdot 4\left(p'^\mu p^\nu + p'^\nu p^\mu - g^{\mu\nu} p\cdot p' - i\epsilon^{\mu\alpha\nu\beta} p'_\alpha p_\beta\right)$$
$$\times \left(k'_\mu k_\nu + k_\mu k'_\nu - g_{\mu\nu} k\cdot k' - i\epsilon_{\mu\rho\nu\sigma} k^\rho k'^\sigma\right)$$

$$= \frac{4g^4}{(p+p')^4}\Big[(p\cdot k')(p'\cdot k) + (p'\cdot k')(p\cdot k) - (p\cdot p')(k\cdot k')$$
$$+ (p\cdot k')(p'\cdot k) + (p'\cdot k')(p\cdot k) - (p\cdot p')(k\cdot k')$$
$$- (p\cdot p')(k\cdot k') - (p\cdot p')(k\cdot k') + g^\mu{}_\mu (p\cdot p')(k\cdot k')$$
$$+ i\epsilon_{\mu\nu\rho\sigma} p'^\mu p^\nu k^\rho k'^\sigma + i\epsilon_{\mu\nu\rho\sigma} p^\mu p'^\nu k^\rho k'^\sigma$$
$$- ig^{\mu\nu}\epsilon_{\mu\nu\rho\sigma} k^\rho k'^\sigma + i\epsilon^{\mu\nu\rho\sigma} k'_\mu k_\nu p'_\rho p_\sigma + i\epsilon^{\mu\nu\rho\sigma} k_\mu k'_\nu p'_\rho p_\sigma$$
$$- ig_{\mu\nu}\epsilon^{\mu\nu\rho\sigma} p'_\rho p_\sigma - \epsilon^{\mu\nu\alpha\beta}\epsilon_{\mu\nu\rho\sigma} p'_\alpha p_\beta k^\rho k'^\sigma\Big]$$

N.B. switched order in $\epsilon_{\mu\alpha\nu\beta}$ to $-\epsilon_{\mu\nu\alpha\beta}$

Now we note $\epsilon_{\mu\nu\rho\sigma}(p'^\mu p^\nu + p^\mu p'^\nu)k^\rho k'^\sigma = 0$ by symmetry $\mu \leftrightarrow \nu$

and $g_{\mu\nu}\epsilon^{\mu\nu\rho\sigma} = 0$ by symmetry of $\epsilon^{\mu\nu\rho\sigma}$.

We also have $\epsilon^{\mu\nu\alpha\beta}\epsilon_{\mu\nu\rho\sigma} = -2(g^\alpha{}_\rho g^\beta{}_\sigma - g^\rho{}_\beta g^\beta{}_\alpha)$

So

$$|\mathcal{M}|^2 = \frac{4g^4}{(p+p')^4}\Big[2(p\cdot k)(p'\cdot k') + 2(p'\cdot k)(p\cdot k') + 2\big((p'\cdot k)(p\cdot k') - (p'\cdot k')(p\cdot k)\big)\Big]$$

$$= \frac{16g^4}{(p+p')^4}(p'\cdot k)(p\cdot k')$$

$\boxed{4.1}$

Now we recall that in the CM frame

$$(p+p')^2 = 4E^2$$

$$p\cdot k' = E^2 + E|\vec{k}|\cos\theta = p'\cdot k$$

So we have

$$|\mathcal{M}|^2 = \frac{16g^4}{16E^4}\left(E^2 + E|\vec{k}|\cos\theta\right)^2$$

$$= \frac{g^4}{E^2}\left(E + \sqrt{E^2 - M_{(\mu)}^2}\cos\theta\right)^2$$

But recall we assumed <u>all</u> fermions ~ massless

$$= g^4(1+\cos\theta)^2$$

There are three other nonvanishing contributions (and twelve vanishing ones)

$|\mathcal{M}|^2 = g^4(1-\cos\theta)^2$ for $e_R^- e_L^+ \to \mu_L^- \mu_R^+$

$|\mathcal{M}|^2 = g^4(1-\cos\theta)^2$ for $e_L^- e_R^+ \to \mu_R^- \mu_L^+$

$|\mathcal{M}|^2 = g^4(1+\cos\theta)^2$ for $e_L^- e_R^+ \to \mu_L^- \mu_R^+$

obtained by $P_R \to P_L$ in muon piece of the trace, which changes sign of one of the $\epsilon^{\mu\nu\alpha\beta}$ terms.

Summing these terms give the differential cross-section

5.

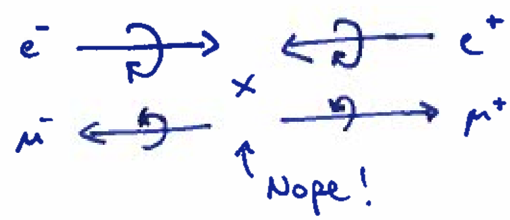
$$\left.\frac{d\sigma}{d\Omega}\right|_{cm} = \frac{\alpha^2}{4E}\left(1+\cos^2\theta\right)$$

← in agreement with our limit for $\frac{m_e}{E}, \frac{m_\mu}{E} \ll 1$

We note that

$$\frac{d\sigma}{d\Omega}\left(e^-_L e^+_L \to \mu^-_R \mu^+_L\right) = \frac{\alpha^2}{4E}\left(1+\cos\theta\right)^2$$

vanishes at $\theta = \pi$. This makes sense, because for back-to-back scattering, the total angular momentum of the final state is opposite to that of the initial state



We can now repeat this calculation with an explicit spinor basis, which is perhaps more intuitive initially, but breaks manifest Lorentz invariance almost from the beginning and is quite painful for the most general case.

Example: polarised $e^+e^- \to \mu^+\mu^-$ (high energy regime)

Recall that we have

$$i\mathcal{M} = \frac{ig^2}{(p+p')^2}\left(\bar{v}(\bar{p}')\gamma^\mu u(p)\right)\left(\bar{u}(k)\gamma^\nu v(\bar{k}')\right)$$

In the high-energy limit, we have

$$u(\bar{p}) = \begin{pmatrix} \sqrt{p\cdot\sigma}\,\xi \\ \sqrt{p\cdot\bar{\sigma}}\,\xi \end{pmatrix} \longrightarrow \sqrt{2E}\begin{pmatrix} \frac{1}{2}(1-\hat{p}\cdot\underline{\sigma})\xi \\ \frac{1}{2}(1+\hat{p}\cdot\underline{\sigma})\xi \end{pmatrix}$$

$$v(\bar{p}) = \begin{pmatrix} \sqrt{p\cdot\sigma}\,\xi \\ -\sqrt{p\cdot\bar{\sigma}}\,\xi \end{pmatrix} \longrightarrow \sqrt{2E}\begin{pmatrix} \frac{1}{2}(1-\hat{p}\cdot\underline{\sigma})\xi \\ -\frac{1}{2}(1+\hat{p}\cdot\underline{\sigma})\xi \end{pmatrix}$$

Now

$$(\hat{p}\cdot\underline{\sigma})\xi_R = +\xi_R$$

$$(\hat{p}\cdot\underline{\sigma})\xi_L = -\xi_L \qquad \leftarrow \text{antiparticles have opposite handedness to their spinors}$$

Let's choose p^μ and p'^μ to have spatial components in the $\pm\hat{z}$-directions. Then for a right-handed electron and left-handed positron

$$\Rightarrow \xi^{e^-} = \begin{pmatrix}1\\0\end{pmatrix} \quad \leftarrow \text{spin up in } \hat{z}\text{-direction}$$

$$\xi^{e^+} = \begin{pmatrix}0\\1\end{pmatrix} \quad \leftarrow \text{spin up in } \hat{z}\text{-direction} \quad \Rightarrow \text{left-handed positron} \Rightarrow \text{right-handed spinor!}$$

So <u>both</u> particles satisfy $(\hat{p}\cdot\underline{\sigma})\xi_R = +\xi_R$

$$u^{e^-}(\bar{p}) = \sqrt{2E}\begin{pmatrix}0\\0\\1\\0\end{pmatrix}$$

$$u^{e^+}(\bar{p}) = \sqrt{2E}\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

So we also have

$$\gamma^0\gamma^\mu = \begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0&\sigma^\mu\\\bar{\sigma}^\mu&0\end{pmatrix} = \begin{pmatrix}\bar{\sigma}^\mu&0\\0&\sigma^\mu\end{pmatrix}$$

This means we get

$$\bar{v}(\bar{p}')\gamma^\mu u(\bar{p}) = \sqrt{2E}\,(0\;0\;0\;-1)\begin{pmatrix}\bar{\sigma}^\mu & 0\\ 0 & \sigma^\mu\end{pmatrix}\sqrt{2E}\begin{pmatrix}0\\0\\1\\0\end{pmatrix}$$

$$= 2E\,(0,-1)\,\sigma^\mu\begin{pmatrix}1\\0\end{pmatrix}$$

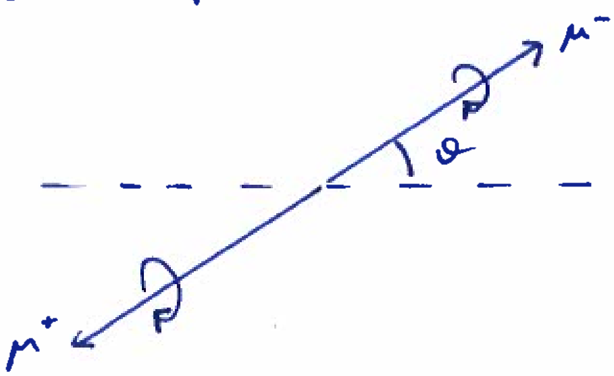
top two × two components vanish

$$= -2E\,(0,1,i,0)$$

$$\mathbb{1} = \begin{pmatrix}1&0\\0&1\end{pmatrix} \quad \sigma^x = \begin{pmatrix}0&1\\1&0\end{pmatrix} \quad \sigma^y = \begin{pmatrix}0&-i\\i&0\end{pmatrix} \quad \sigma^z = \begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

N.B. The interpretation of this is that the exchanged photon has circular polarisation
$$\epsilon_+ = \frac{1}{\sqrt{2}}(\hat{x} + i\hat{y})$$

We also need the muon half of the matrix element. Let's work with the case in which its right-handed, so that the μ^+ is left-handed. Then our final states look like



To calculate this part of the matrix element, we use a neat trick: recall that $\bar{\psi}\gamma^\mu\psi$ transforms like a 4-vector (and we're assuming both $m_e = m_\mu = 0$) so we can just rotate our electron result

$$\bar{u}(k)\gamma^\mu v(k') = \left(\bar{v}(k')\gamma^\mu u(k)\right)^*$$

$$= \left[-2E(0, \cos\theta, i, -\sin\theta)\right]^*$$

$$= -2E(0, \cos\theta, -i, -\sin\theta)$$

Since this vector can be interpreted as the polarisation of the virtual photon, we will have a nonzero amplitude when this has nonzero overlap with $-2E(0,1,i,0)$.

We now plug these into

$$i\mathcal{M} = \frac{ig^2}{(p+p')^2}(\bar{v}\gamma^\mu u)(\bar{u}\gamma_\mu v)$$

$$= \frac{ig^2}{(p+p')^2}\left(-2E(0,1,i\ 0)\cdot -2E\begin{pmatrix}0\\ \cos\theta\\ -i\\ -\sin\theta\end{pmatrix}\right)$$

$$= \frac{ig^2 4E^2}{4E^2}(-\cos\theta - 1)$$

$$= -ig^2(1+\cos\theta)$$

$\Rightarrow |\mathcal{M}|^2 = g^4(1+\cos\theta)^2$ – in agreement with our previous result (p. 138) ✓

Similar approaches will give the combinations of $e^{+/-}_{L/R}$ and $\mu^{+/-}_{L/R}$, as before.

<u>Example: unpolarised $e^+e^- \to \mu^+\mu^-$ (low energy regime)</u>

$\boxed{\text{P+S } 5.31}$

Recall our old result for unpolarised scattering

$$\left.\frac{d\sigma}{d\Omega}\right|_{cm} = \frac{\alpha^2}{16E^2}\sqrt{1-\frac{m_\mu^2}{E^2}}\left(1+\frac{m_\mu^2}{E^2}+\left(1-\frac{m_\mu^2}{E^2}\right)\cos^2\theta\right) \quad (E_{cm}=2E)$$

$$\xrightarrow[|\vec{k}|\to 0]{} \frac{\alpha^2}{2E_{cm}^2}\sqrt{1-\frac{m_\mu^2}{E^2}} = \frac{\alpha^2}{2E_{cm}^2}\frac{|\vec{k}|}{E}$$

We can also derive this from an explicit expression for our spinors. We assume $m_e \ll E \ll m_\mu$. Then we can assume the $e^{-/+}$ have definite helicity. Let's work with our choices from before, so

$$u(\bar{p}) = \begin{pmatrix} \sqrt{p\cdot\sigma}\,\xi \\ \sqrt{p\cdot\bar{\sigma}}\,\xi \end{pmatrix} \rightarrow \sqrt{2E}\begin{pmatrix} \frac{1}{2}(1-\hat{p}\cdot\underline{\sigma})\xi \\ \frac{1}{2}(1+\hat{p}\cdot\underline{\sigma})\xi \end{pmatrix}$$

and $\bar{v}(\bar{p}')\gamma^\mu u(\bar{p}) = -2E(0,1,i,0)$

$$v(\bar{p}) = \begin{pmatrix} \sqrt{p\cdot\sigma}\,\xi \\ -\sqrt{p\cdot\bar{\sigma}}\,\xi \end{pmatrix} \rightarrow \sqrt{2E}\begin{pmatrix} \frac{1}{2}(1-\hat{p}\cdot\underline{\sigma})\xi \\ -\frac{1}{2}(1+\hat{p}\cdot\underline{\sigma})\xi \end{pmatrix}$$

But the muons are taken to be nonrelativistic, so

$$u(\bar{p}) = \begin{pmatrix} \sqrt{p\cdot\sigma}\,\xi \\ \sqrt{p\cdot\bar{\sigma}}\,\xi \end{pmatrix} \rightarrow \sqrt{m}\begin{pmatrix} \xi \\ \xi \end{pmatrix}$$

$$v(\bar{p}) = \begin{pmatrix} \sqrt{p\cdot\sigma}\,\xi \\ -\sqrt{p\cdot\bar{\sigma}}\,\xi \end{pmatrix} \rightarrow \sqrt{m}\begin{pmatrix} \xi \\ -\xi \end{pmatrix}$$

$$\Rightarrow \bar{u}(\bar{k})\gamma^\mu v(\bar{k}') = m(\xi^\dagger, \xi^\dagger)\begin{pmatrix} \bar{\sigma}^\mu & 0 \\ 0 & \sigma^\mu \end{pmatrix}\begin{pmatrix} \xi' \\ -\xi' \end{pmatrix} = \begin{cases} 0 & \mu = 0 \\ -2m\,\xi^\dagger\sigma^i\xi' & \mu = i \end{cases}$$

Thus
$$i\mathcal{M} = \frac{ig^2}{(p+p')^2}\left(\bar{v}(\bar{p}')\gamma^\mu u(\bar{p})\right)\left(\bar{u}(\bar{k})\gamma_\mu v(\bar{k}')\right)$$

N.B. $E \approx m$

$$= \frac{ig^2}{4m^2}(-2E)(0,1,i,0)\begin{pmatrix} 0 \\ -2m\,\xi^\dagger\sigma^x\xi' \\ -2m\,\xi^\dagger\sigma^y\xi' \\ -2m\,\xi^\dagger\sigma^z\xi' \end{pmatrix}$$

$$= ig^2\frac{(-2E)(-2m)}{4m^2}\left(\xi^\dagger(\sigma^x + i\sigma^y)\xi'\right)$$

$$= ig^2(-2)\,\xi^\dagger\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\xi'$$

$$\left.\begin{aligned} \sigma^x &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ \sigma^y &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \end{aligned}\right\} \sigma^x + i\sigma^y = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}$$

There is no angular dependence at all

$\Rightarrow$ $\mu^{+/-}$ emitted in an s-wave (no orbital angular momentum)

$\Rightarrow$ since the $e^{+/-}$ had spin 1 in $\hat{z}$-direction this means both $\mu^{+/-}$ have to have spin up in $\hat{z}$-direction

Now we can sum over final state spins

$$iM = ig^2(-2)\left[(1,0)\begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} + (1,0)\begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}\right] = -2ig^4$$

$$\Rightarrow |M|^2 = 4g^4$$

The cross-section is

$$\left.\frac{d\sigma}{d\Omega}\right|_{cm}(e_R^- e_R^+ \to \mu^+\mu^-) = \frac{1}{2E_{cm}^2}\frac{|\bar{k}|}{16\pi^2 E_{cm}}|M|^2$$

$$= \frac{1}{2E_{cm}^2}\frac{|\bar{k}|}{E_{cm}}\frac{4g^4}{(4\pi)^2}$$

Averaging over initial states and noting that the same expression holds for $e_L^- e_R^+ \to \mu^+\mu^-$, we obtain

$$\left.\frac{d\sigma}{d\Omega}\right|_{cm}(e^-e^+ \to \mu^+\mu^-) = \frac{1}{4}\times 2\times\frac{1}{E_{cm}^2}\frac{|\bar{k}|}{E}\alpha^2 = \frac{\alpha^2}{2E_{cm}}\frac{|\bar{k}|}{E}$$

exactly as we had before!