Quantum Field Theory I: PHYS 721
Problem Set 3

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The questions in this problem give you some practice at manipulating spinors and summarising how particles and fields relate.

Question 1 — 10pts

(a) Prove the spinor relation

$$\overline{u}^r(\vec{p})\gamma^\mu u^s(\vec{q}) = \frac{1}{2m}\overline{u}^r(\vec{p})\left[p^\mu + q^\mu + i\sigma^{\mu\nu}(p_\nu - q_\nu)\right]u^s(\vec{q}).$$

where $\sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/2$. This relation is known as the "Gordon identity". Note that this is named after Walter Gordon of Klein-Gordon equation fame, and not Paul Gordan of Clebsch-Gordan coefficient fame.

(b) Using the identity

$$(\sigma^\mu)_{\alpha\beta}(\sigma_\mu)_{\gamma\delta} = 2\epsilon_{\alpha\gamma}\epsilon_{\beta\delta},$$

where $\epsilon_{12} = +1$, show that

$$\left[\overline{u}_1\gamma^\mu\left(\frac{1+\gamma^5}{2}\right)u_2\right]\left[\overline{u}_3\gamma_\mu\left(\frac{1+\gamma^5}{2}\right)u_4\right] = -\left[\overline{u}_1\gamma^\mu\left(\frac{1+\gamma^5}{2}\right)u_4\right]\left[\overline{u}_3\gamma_\mu\left(\frac{1+\gamma^5}{2}\right)u_2\right].$$

Here $\overline{u}_{1,3}$ and $u_{2,4}$ are four different spinors. This is an example of a "Fierz identity". These identities relate products of spinor bilinears to sums of products of more useful spinor bilinears.

Both the Gordon and various Fierz identities are often used in calculations of scattering amplitudes involving fermions.

Question 2 — 10pts

Write 250 to 300 words[1] discussing the relationship between particles and fields in quantum field theory. You should illustrate this with at least two examples. Your response should be written in full sentences and addressed to other graduate students who have taken QFT, but don't use it regularly in their research and perhaps have forgotten some key pieces. You will be graded using the following rubric:

[1] I will count.

Aspect	**Points**	**If you:**
Physics	4	Correctly characterise: Lorentz and Poincaré groups; representations of those two groups; relation between particles and the Poincaré group, and fields and the Lorentz group.
	2	Correctly characterise most of these points, but not all; or describe all points, but miss key information.
	0	Completely misconstrue the relationships.
Examples	4	Provide at least two examples.
	2	Provide one example; or two examples, but not correctly.
	0	Give no examples
Audience	2	Correctly gauge the understanding of the audience, including defining and illustrating terms as appropriate.
	1	Give a too-technical or too-simplistic explanation.